Factorial

Factorial: The factorial of a natural number n is the product of all natural numbers less than or equal to n and it is denoted by $n!$ or $\underline{|n}$.

i.e. $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$

e.g. $4! = 4 \times 3 \times 2 \times 1 = 24$

Also the value of $0!$ is 1, according to the connection for an empty product. i.e. $0! = 1$

Note :→ Factorials of negative integers are undefined.

(#) $0! = 1$

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880$

$10! = 10 \times 9! = 362880$

Note: $8! = 8 \times 7! = 8 \times 7 \times 6! = 8 \times 7 \times 6 \times 5!$

Q.1. Evaluate the following:

(i) $3! + 2!$

(ii) $(3+2)!$

(iii) $3 + 2!$

(iv) $3! + 2$

(v) $\lfloor\underline{3} + \lfloor\underline{4}$

(vi) $\lfloor\underline{5} - \lfloor\underline{3}$

(vii) $(5-3)!$

(viii) $5 - 3!$

(ix) $6! - 4!$

(x) $\dfrac{6!}{3!}$

(xi) $\left(\dfrac{6}{3}\right)!$

(xii) $3! \times 2!$

(xiii) $(3 \times 2)!$

(xiv) $(-5)!$

(xv) $-5!$

(xvi) $2! \times 0!$

(xvii) $\dfrac{50!}{48!}$

(xviii) $\dfrac{6!}{4! \times 2!}$

(xix) $\dfrac{8!}{4! \times 3!}$

(xx) $\dfrac{102!}{100!}$

(xxi) $\dfrac{70!}{3! \times 67!}$

(xxii) $\dfrac{10!}{5! \times 2!}$

(xxiii) $\dfrac{n!}{(n-2)!}$

(xxiv) $\dfrac{(n+1)!}{(n-1)!}$

Q.2. If n is a natural number, then solve the following equations:

(i) $(n+1)! = 12.(n-1)!$

(ii) $(n+2)! = 60.(n-1)!$

(iii) $\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$

(iv) $\frac{1}{4!} + \frac{1}{5!} = \frac{n}{6!}$

(v) $\frac{1}{6!} + \frac{1}{7!} = \frac{n}{8!}$

Answers 1.

1. (i) $3! + 2! = 3\times2\times1 + 2\times1$
$= 6 + 2 = 8$

(ii) $(3+2)! = 5! = 5\times4\times3\times2\times1 = 120$

Conclusion: $m! + n! \neq (m+n)!$

(iii) $3 + 2! = 3 + 2\times1 = 3 + 2 = 5$

(iv) $3! + 2 = 3\times2\times1 + 2 = 6 + 2 = 8$

(v) $\underline{|3} + \underline{|4} = 3\times2\times1 + 4\times3\times2\times1$
$= 6 + 24 = 30$

(vi) $\underline{|5} - \underline{|3} = 5\times4\times3\times2\times1 - 3\times2\times1$
$= 120 - 6 = 114$

(vii) $(5-3)! = 2! = 2\times1 = 2$.

Conclusion: $m! - n! \neq (m-n)!$

(viii) $5 - 3! = 5 - 3\times2\times1 = 5 - 6 = -1$

(ix) $6! - 4! = 6\times5\times4\times3\times2\times1 - 4\times3\times2\times1$
$= 720 - 24 = 696$

(x) $\dfrac{6!}{3!} = \dfrac{6\times5\times4\times\cancel{3!}}{\cancel{3!}} = 120$

(xi) $\left(\frac{6}{3}\right)! = 2! = 2 \times 1 = 2$

Conclusion: $\left(\frac{m}{n}\right)! \neq \frac{m!}{n!}$

(xii) $3! \times 2! = 3 \times 2 \times 1 \times 2 \times 1 = 12$

(xiii) $(3 \times 2)! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Conclusion: $m! \times n! \neq (m \times n)!$

(xiv) $(-5)! \rightarrow$ undefined

(xv) $-5! = -(5 \times 4 \times 3 \times 2 \times 1) = -120$

(xvi) $2! \times 0! = 2 \times 1 \times 1 = 2$

(xvii) $\frac{50!}{48!} = \frac{50 \times 49 \times \cancel{48!}}{\cancel{48!}} = 2450$

(xviii) $\frac{6!}{4! \times 2!} = \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1} = 15$

or $\frac{6!}{4! \times 2!} = \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2} \times 1} = 15$

(xix) $\frac{8!}{4! \times 3!} = \frac{8 \times 7 \times \overset{2}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2} \times 1} = 280$

(xx) $\frac{102!}{100!} = \frac{102 \times 101 \times \cancel{100!}}{\cancel{100!}} = 10302$

(xxi) $\dfrac{70!}{3!\times 67!} = \dfrac{70\times \overset{23}{\cancel{69}}\times \overset{34}{\cancel{68}}\times \cancel{67!}}{\cancel{3}\times\cancel{2}\times 1\times\cancel{67!}} = 54740$

(xxii) $\dfrac{10!}{5!\times 2!} = \dfrac{10\times 9\times 8\times 7\times \overset{3}{\cancel{6}}\times\cancel{5!}}{\cancel{5!}\times\cancel{2}\times 1} = 15120$

(xxiii) $\dfrac{n!}{(n-2)!} = \dfrac{n.(n-1)\cdot\cancel{(n-2)!}}{\cancel{(n-2)!}} = n^2-n$

(xxiv) $\dfrac{(n+1)!}{(n-1)!} = \dfrac{(n+1).n.\cancel{(n-1)!}}{\cancel{(n-1)!}} = n^2+n$

Answer 2.

(i) Given that

$$(n+1)! = 12.(n-1)!$$

$$\Rightarrow (n+1).n.\cancel{(n-1)!} = 12.\cancel{(n-1)!}$$

$$\Rightarrow (n+1).n = 4\times 3$$

$$\Rightarrow \boxed{n=3}$$

Or $$(n+1)! = 12.(n-1)!$$

$$\Rightarrow (n+1).n.\cancel{(n-1)!} = 12.\cancel{(n-1)!}$$

$$\Rightarrow n^2+n = 12$$

$$\Rightarrow n^2+n-12 = 0$$

$$\Rightarrow n^2+4n-3n-12 = 0$$

$$\Rightarrow n(n+4)-3(n+4) = 0$$

$$\Rightarrow (n+4)(n-3) = 0$$

$\Rightarrow$ either $n+4=0$ or $n-3=0$

$\Rightarrow$ either $n=-4$ or $n=3$

Rejecting $n=-4$

$\therefore \boxed{n=3}$

(ii) Given $(n+2)! = 60.(n-1)!$

4	2	60
	2	30
	3	15
	5	5
		1

$$\Rightarrow (n+2).(n+1).n.\cancel{(n-1)!} = 60.\cancel{(n-1)!}$$

$$\Rightarrow (n+2).(n+1).n = 5\times 4\times 3$$

$$\Rightarrow \boxed{n=3}$$

(iii) Given that

$$\frac{1}{9!}+\frac{1}{10!}=\frac{n}{11!}$$

$$\Rightarrow \frac{1}{9!}+\frac{1}{10\times 9!}=\frac{n}{11\times 10\times 9!}$$

$$\Rightarrow \frac{1}{9!}\left[\frac{1}{1}+\frac{1}{10}\right]=\frac{n}{11\times 10\times 9!}$$

$$\Rightarrow \left[\frac{1}{1}+\frac{1}{10}\right]=\frac{n\times \cancel{9!}}{11\times 10\times \cancel{9!}}$$

$$\Rightarrow \frac{10+1}{10}=\frac{n}{110}$$

$$\Rightarrow \frac{11}{10}=\frac{n}{110}$$

$$\Rightarrow \frac{11}{\cancel{10}}\times 11\cancel{0}=n$$

$$\Rightarrow 121=n$$

i.e. $\boxed{n=121}$

(iv) Given that $\frac{1}{4!}+\frac{1}{5!}=\frac{n}{6!}$

$$\Rightarrow \frac{1}{4!}+\frac{1}{5\times 4!}=\frac{n}{6\times 5\times 4!}$$

$$\Rightarrow \frac{1}{4!}\left[\frac{1}{1}+\frac{1}{5}\right]=\frac{n}{30\times 4!}$$

$$\Rightarrow \left[\frac{5+1}{5}\right]=\frac{n\times \cancel{4!}}{30\times \cancel{4!}}$$

$$\Rightarrow \frac{6}{5}=\frac{n}{30}$$

$$\Rightarrow \frac{6}{\cancel{5}} \times \cancel{30}^{6} = n$$

$$\Rightarrow 36 = n$$

i.e. $\boxed{n=36}$

(v) $$\frac{1}{6!} + \frac{1}{7!} = \frac{n}{8!}$$

$$\Rightarrow \frac{1}{6!} + \frac{1}{7\times 6!} = \frac{n}{8\times 7\times 6!}$$

$$\Rightarrow \frac{1}{6!}\left[\frac{1}{1} + \frac{1}{7}\right] = \frac{n}{56\times 6!}$$

$$\Rightarrow \frac{1}{6!}\left[\frac{7+1}{7}\right] = \frac{n}{56\times 6!}$$

$$\Rightarrow \frac{8}{7} = \frac{n\times \cancel{6!}}{56\times \cancel{6!}}$$

$$\Rightarrow \frac{8}{\cancel{7}} \times \cancel{56}^{8} = n$$

$$\Rightarrow 64 = n$$

i.e. $\boxed{n=64}$

Permutations & Combinations

Fundamental Principles of Counting:

(i) Fundamental Principle of Multiplication: If an event can occur in m different ways and if following it, a second event can occur in n different ways, then the two events in succession can occur in $m \times n$ different ways.

(ii) Fundamental Principle of Addition: If two events can occur independently in exactly m ways and n ways respectively, then either of the two events can occur in $(m+n)$ ways.

e.g. there are 3 candidates for a Classical, 5 for a Mathematical and 4 for a Natural Science scholarship.

Q.(1) In how many ways can these scholarships be awarded?

Q.(2) In how many ways one of these scholarships be awarded?

Ans(1). Classical scholarship can be awarded to any one of the three candidates. So, there are three ways of awarding it. Similarly, Mathematical and Natural Science scholarships can be awarded in 5 and 4 ways respectively. So, total number of ways of awarding three scholarships $= 3 \times 5 \times 4 = 60$.

Ans(2) No. of ways of awarding one of the three scholarships

$= 3 + 5 + 4 = 12$.

Arrangement → Permutations → P → ^nP_r ; $n \geq r \geq 0$

Selection → Combination → C → ^nC_r ; $n \geq r \geq 0$

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^nC_r = \frac{n!}{(n-r)! \times r!}$$

Arrangement :- 3 balls , arrange 2 balls

B1, B2, B3

B1 B2, B1 B3, B2 B1, B3 B1, B2 B3, B3 B2

$n = 3$, $r = 2$

Total no. of arrangements $= {}^3P_2 = \frac{3!}{(3-2)!}$

$$= \frac{3!}{1!} = \frac{3 \times 2 \times 1}{1} = 6$$

Selection 3 balls , select 2 balls

B1, B2, B3

B1 B2 , B1 B3, B2 B3

$n = 3$
$r = 2$

No. of ways of selecting 2 balls out of 3 balls

$$= {}^3C_2 = \frac{3!}{(3-2)! \times 2!} = \frac{3!}{1! \times 2!}$$

$$= \frac{3 \times \cancel{2} \times 1}{1 \times \cancel{2} \times 1} = 3$$

Permutation $\rightarrow P \rightarrow {}^nP_r$; $n \geq r \geq 0$
(Arrangement)

Combination $\rightarrow C \rightarrow {}^nC_r$; $n \geq r \geq 0$
(Selection)

${}^nP_r$	${}^nC_r$

$${}^nP_r = \frac{n!}{(n-r)!}$$

$${}^nC_r = \frac{n!}{(n-r)! \times r!}$$

5 toh times

Note:
$(n-r)!$ ✓
$n-r!$ ✗
$(n-r!)$ ✗
$n!-r!$ ✗
$n+r!$ ✗

eg. $${}^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = \frac{5\times4\times3\times2\times1}{1} = 120$$

$${}^6C_4 = \frac{6!}{(6-4)! \times 4!} = \frac{6!}{2! \times 4!} = \frac{\overset{3}{\cancel{6}}\times5\times\cancel{4!}}{\cancel{2}\times1\times\cancel{4!}} = 15$$

eg. $${}^4C_0 = \frac{4!}{(4-0)! \times 0!} = \frac{\cancel{4!}}{\cancel{4!}\times1} = 1$$

$${}^4C_4 = \frac{4!}{(4-4)! \times 4!} = \frac{\cancel{4!}}{0! \times \cancel{4!}} = \frac{1}{1} = 1$$

$${}^4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4\times3\times2\times1}{1} = 24$$

$${}^4P_0 = \frac{4!}{(4-0)!} = \frac{\cancel{4!}^{\,1}}{\cancel{4!}} = 1$$

#

3 balls

B_1, B_2, B_3

arrange 2 balls

$B_1 \ B_2$
$B_2 \ B_1$
$B_1 \ B_3$
$B_3 \ B_1$
$B_2 \ B_3$
$B_3 \ B_2$

6 arrangements

$n = 3$
$r = 2$

No. of ways to arrange 2 balls at two places out of 3 balls $= {}^3P_2 = \frac{3!}{(3-2)!} = \frac{3\times2\times1}{1!} = \frac{6}{1} = 6$

26 alph. arrange 2 alph.

$${}^{26}P_2 = \frac{26!}{(26-2)!} = \frac{26!}{24!} = \frac{26\times25\times\cancel{24!}}{\cancel{24!}} = 650$$

3 balls

B_1, B_2, B_3

Select any 2 balls

B_1B_2, B_1B_3, B_2B_3

3 ways

Ram
Syam
Balram

No. of ways of selecting 2 balls out of 3 balls

$$= {}^3C_2 = \frac{3!}{(3-2)!\times2!} = \frac{3!}{1!\times2!}$$

$$= \frac{3\times\cancel{2!}}{1\times\cancel{2!}} = 3$$

26 alph.

select 2 alph.

$${}^{26}C_2 = \frac{26!}{24!\times2!} = \frac{{}^{13}\cancel{26}\times25\times\cancel{24!}}{\cancel{24!}\times\cancel{2}\times1} = 325$$

Q. ${}^5C_0$, ${}^5C_1$, ${}^5C_2$, ${}^5C_3$, ${}^5C_4$, ${}^5C_5$, ${}^7C_4$, ${}^6C_3$, ${}^4C_0$, ${}^8C_8$, ${}^6C_1$, ${}^7C_3$, ${}^3C_2$, ${}^3C_0$, ${}^4C_4$

(xi) $${}^7C_7 = \frac{7!}{(7-7)! \times 7!} = \frac{\cancel{7!}}{0! \times \cancel{7!}} = \frac{1}{1} = 1$$

(x) $${}^7C_0 = \frac{7!}{(7-0)! \times 0!} = \frac{\cancel{7!}}{\cancel{7!} \times 1} = 1$$

Q. If ${}^nP_4 = 20 \times {}^nP_2$, find n

Ans Given that

$${}^nP_4 = 20 \times {}^nP_2$$

$$\Rightarrow \frac{n!}{(n-4)!} = 20 \times \frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{\cancel{n!}}{(n-4)!} \times \frac{(n-2)!}{\cancel{n!}} = 20$$

$$\Rightarrow \frac{(n-2)!}{(n-4)!} = 20$$

$$\Rightarrow \frac{(n-2) \times (n-3) \times \cancel{(n-4)!}}{\cancel{(n-4)!}} = 20$$

$$\Rightarrow n^2 - 3n - 2n + 6 = 20$$

$$\Rightarrow n^2 - 5n + 6 - 20 = 0$$

$$\Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$

$$\Rightarrow n(n-7) + 2(n-7) = 0$$

$$\Rightarrow (n-7)(n+2) = 0$$

$$\Rightarrow n-7=0 \text{ or } n+2=0$$

$$\Rightarrow n = 7 \text{ or } n = -2$$

Rejecting $n = -2$

$$\therefore \boxed{n = 7}$$

Ans 9(i) Given that

$^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$

$^nP_r = \dfrac{n!}{(n-r)!}$

$\Rightarrow \dfrac{^{2n+1}P_{n-1}}{^{2n-1}P_n} = \dfrac{3}{5}$

$\Rightarrow \dfrac{\dfrac{(2n+1)!}{(2n+1-n+1)!}}{\dfrac{(2n-1)!}{(2n-1-n)!}} = \dfrac{3}{5}$

$\Rightarrow \dfrac{\dfrac{(2n+1)!}{(n+2)!}}{\dfrac{(2n-1)!}{(n-1)!}} = \dfrac{3}{5}$

$\Rightarrow \dfrac{(2n+1)!}{(n+2)!} \times \dfrac{(n-1)!}{(2n-1)!} = \dfrac{3}{5}$

$\Rightarrow \dfrac{(2n+1)\times(2\cancel{n})\times\cancel{(2n-1)!}\times\cancel{(n-1)!}}{(n+2)\times(n+1)\times\cancel{n}\times\cancel{(n-1)!}\times\cancel{(2n-1)!}} = \dfrac{3}{5}$

$\Rightarrow \dfrac{4n+2}{n^2+n+2n+2} = \dfrac{3}{5}$

$\Rightarrow \dfrac{4n+2}{n^2+3n+2} = \dfrac{3}{5}$

$\Rightarrow 20n+10 = 3n^2+9n+6$

$\Rightarrow 0 = 3n^2+9n+6-20n-10$

$\Rightarrow 0 = 3n^2-11n-4$

i.e $3n^2-11n-4=0$

$\Rightarrow 3n^2-12n+n-4=0$

$\Rightarrow 3n(n-4)+1(n-4)=0$

$\Rightarrow (n-4)(3n+1)=0$

$\Rightarrow n-4=0$ or $3n+1=0$

$\Rightarrow n=4$ or $n=-\dfrac{1}{3}$

Rejecting $n=-\dfrac{1}{3}$

$\therefore n=4$

Ans 10. No. of teachers = 30

P	V.P.
30 ×	29

V.P. P

$$^{30}P_2 = \frac{30!}{(30-2)!} = \frac{30!}{28!}$$

$$= \frac{30 \times 29 \times 28!}{28!} = 870$$

Ans 11. 5 flags G, B, R, Y, V — 325

1 flag	2 flags			3 flags	4 flags	5 flags
G	GB	BR	RV			
B	BG	RB	VR			
R	GR	BY	YV			
Y	RG	YB	VY			
V	GY	BV				
	YG	VB				
	GV	RY				
	VG	YR				
5	20			$^5P_3 = 60$	$^5P_4 = 120$	$^5P_5 = 120$
$^5P_1 = 5$	$^5P_2 = 20$					

Total no. of signals $= {^5P_1} + {^5P_2} + {^5P_3} + {^5P_4} + {^5P_4}$

$= 5 + 20 + 60 + 120 + 120$

$= 325$

Ans 12. TRIANGLE letters

8	7	6	5	4	3	2	1

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$

$$^8P_8 = \frac{8!}{0!} = \frac{8!}{1} = 8! = 40320$$

T	.	.	.	.	.	.	E

$1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ 1

$$^6P_6 = \frac{6!}{0!} = \frac{720}{1} = 720$$